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An Isotropic Metamaterial Formed with Ferroelectric Ceramic Spherical Inclusions

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Abstract—A metamaterial that is formed by two types of spherical particles of different diameters has been considered. The particles are prepared from a ferroelectric ceramic material with a high permittivity ($\varepsilon \geq 400$), so that the electromagnetic field inside the particles obeys the electrodynamic laws and the external field can be considered in the electro- or magnetostatic approximation. The dependences of the properties of the structure under investigation on the parameters of the ferroelectric ceramic material and on the sizes of the particles are analyzed. The results of the experimental investigation of the ferroelectric ceramic resonators are reported.

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1. INTRODUCTION

A great interest has been expressed to a new class of materials, the so-called metamaterials [1–4], which, within a certain frequency range, possess unique properties, in particular, a negative magnetic permeability and a negative permittivity. Metamaterials are artificial composite media consisting of dielectric or conductive components that form a regular structure characterized by a negative effective permittivity and a negative effective permeability (ε and μ, respectively) and, accordingly, by a negative refractive index. In the majority of cases, such artificial materials are formed by resonant particles. There is a negative dispersion observed in metamaterials, and the propagating wave is backward. The properties of metamaterials are exhibited in a limited frequency range [5]. These materials can be used as a basis for the design of unique devices [6], such as flat electromagnetic lenses having no diffraction limit (superlenses) [7], invisibility cloaks [8–10], etc., which raises a burgeoning interest in their practical application.

2. ISOTROPIC METAMATERIAL

The idea of using spherical dielectric resonators was first reported by Holloway and Kuester [11]. In this case, the metamaterial-forming particles were considered to be resonators prepared from a material exhibiting simultaneously high values of the permittivity and the magnetic permeability (such materials are unfeasible for manufacturing). In 2004, a model was proposed for an isotropic metamaterial that would consist of ferroelectric spherical particles supporting the H_{111} and E_{111} resonant modes [12, 13]. This model is applicable for the real manufacturing of the structure, because spherical particles in this case were assumed to be made of the material with a high value of the permittivity $\varepsilon = 400–1000$, whereas the magnetic permeability of this material could be equal to that in vacuum. High values of the permittivity are necessary for supporting resonance in the spherical components, provided the spherical particle sizes are considerably less than the wavelength in the surrounding medium. A ferroelectric ceramic material having a high permittivity and a reasonably low level of loss was chosen as the material for manufacturing such spherical particles.

Let us consider a structure containing two sublattices of dielectric spheres in which the H_{111} and E_{111} resonant modes are excited. We assume that the sublattices of spherical particles with different radii are arranged as the NaCl structure (Fig. 1). The unit cell

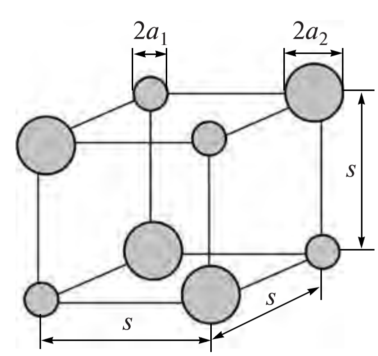

Fig. 1. A model of the isotropic metamaterial as a structure of the NaCl molecule.

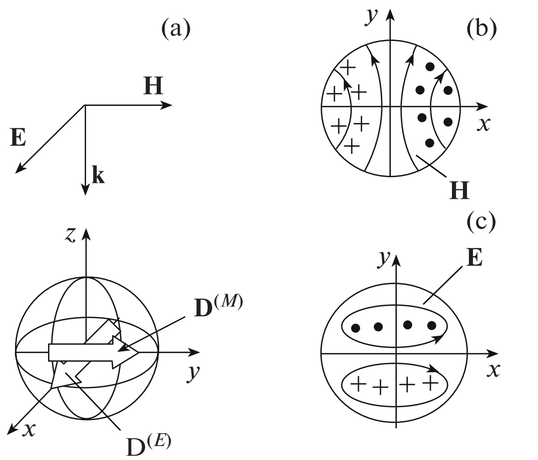

Fig. 2. (a) Plane electromagnetic wave and the induced electric and magnetic dipoles inside a spherical resonator and (b, c) distribution of electromagnetic fields for resonances with (b) magnetic and (c) electric fields inside the resonator.

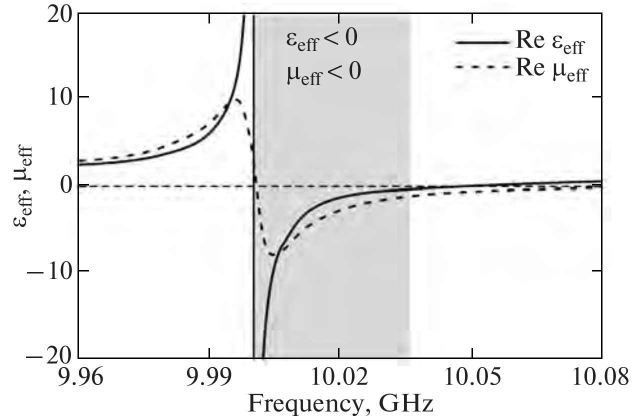

Fig. 3. Frequency dependences of the effective permittivity and the effective permeability of the metamaterial for resonators with the radii $a_1 = 748$ mm and $a_2 = 1.069$ mm, the lattice spacing $s = 4$ mm, the permittivity of the resonators $\varepsilon_1 = 400$, and $\tan\delta = 10^{-3}$ and the permittivity of the medium ($\varepsilon_2 = 1$) in which the resonators are embedded.

size is $s \geq 4(a_1 + a_2)$). The radii of the spheres are chosen from the equality condition for resonance frequencies of the corresponding resonant modes.

Let us consider diffraction by a spherical particle of a plane electromagnetic wave with the electric field amplitude E_0, which is linearly polarized along the x axis. The wave propagates along the z axis in the medium with the permittivity ε_2 and the wave number k_2 (Fig. 2).

Solving the diffraction problem [14–17] makes it possible to determine the distribution of the electromagnetic field inside spherical resonators $\mathbf{E}^{(t)}(r, \theta, \varphi)$. The distribution of the electric and magnetic components for resonances with magnetic and electric fields is displayed in Figs. 2b and 2c, respectively. Integrating the fields over the spherical volume yields the electric dipole moment of the spherical particle along the x axis and the magnetic dipole moment of the spherical particle along the y axis (Fig. 2a).

The averaged macroscopic magnetization and the averaged dielectric polarization can be found as the corresponding dipole moment divided by the volume taken by the unit cell occupied by the particles. Accordingly, the effective permittivity and the effective permeability of the medium take the following form:

$$\varepsilon_{eff}(\omega) = \frac{4}{3}\pi a_2^3 \frac{2}{s^3}\varepsilon_1 b_1^{(t)}(k_1 a_2) I(k_1 a_2), \quad (1)$$

$$\mu_{eff}(\omega) = \frac{4}{3}\pi a_1^3 \frac{2}{s^3}\sqrt{\varepsilon_1} a_1^{(t)}(k_1 a_1) I(k_1 a_1), \quad (2)$$

where a_1 and a_2 are the radii of spherical particles ($a_2 > a_1$), $a_1^{(t)}$ and $b_1^{(t)}$ are the amplitudes of the spherical wave functions, ε_1 is the permittivity of the particle material, and $I(\xi)$ is the result of integration over the volume of the particles.

The frequency dependences of the effective permittivity $\varepsilon_{eff}(\omega)$ and the effective permeability $\mu_{eff}(\omega)$ are determined by the frequency dependences of the amplitudes $a_1^{(t)}$ and $b_1^{(t)}$ of the modes excited in the spherical particles: the H_{111} mode in the particles of a smaller radius a_1 and the E_{111} mode in the particles of a larger radius a_2. The radii a_1 and a_2 are chosen from the equality condition for resonance frequencies of the two resonant modes.

Figure 3 displays the frequency dependences of the effective permittivity $\varepsilon_{eff}(\omega)$ and the effective permeability $\mu_{eff}(\omega)$ [16].

In the shaded region, both the permittivity and the permeability are negative, which is accompanied with a negative dispersion of the medium. The electromagnetic wave propagating in this medium is backward.

3. REQUIREMENTS TO THE PROPERTIES OF THE DIELECTRIC MATERIAL

The dependence of the properties of the engineered metamaterial on the material parameters of the spherical resonators is of great importance. These material parameters can be the permittivity and the level of loss. The calculations have demonstrated that, as the permittivity increases, the width of the frequency range in which the artificial medium exhibits the required properties decreases. Moreover, an increase in the loss in the ferroelectric material also leads to a narrowing of the operating frequency band (Fig. 4). It is evident that, at a sufficiently high level of loss, the metamate-

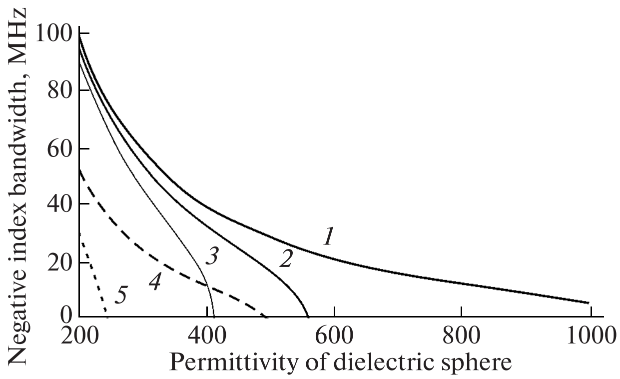

Fig. 4. Bandwidth of the operating frequency range with a negative dispersion as a function of the permittivity of the particle material for central operating frequencies of (*1–3*) 10 and (*4*, *5*) 15 GHz at the levels of loss $\tan\delta = (1, 4)$ 0.001, (*2*, *5*) 0.003, and (*3*) 0.005.

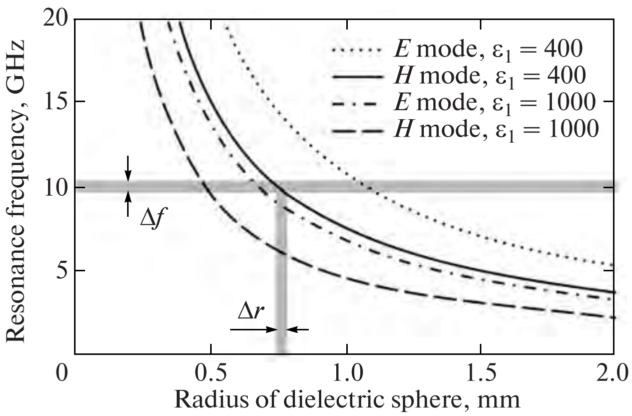

Fig. 5. Resonance frequency as a function of the spherical resonator radius for $\varepsilon_1 = 400$ and 1000. Highlighted are the ranges of $\Delta f = 32$ MHz and $\Delta r = 3$ μm.

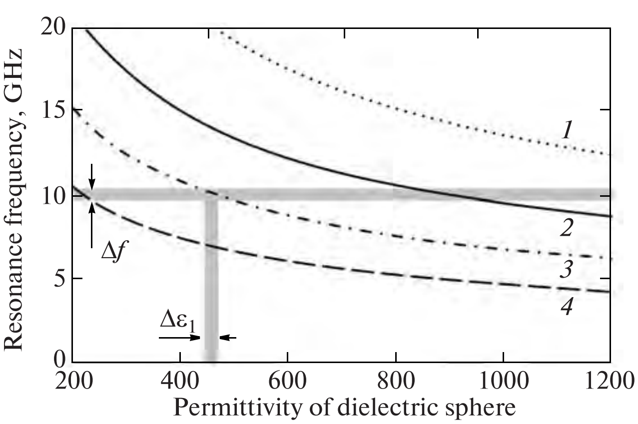

Fig. 6. Resonance frequency as a function of the permittivity of the material of spherical resonators with radii $r =$ (*1*, *2*) 0.5 and (*3*, *4*) 1.0 mm. Highlighted are the ranges of $\Delta f = 32$ MHz and $\Delta\varepsilon_1 = 5$. Curves *1*, *3* and *2*, *4* correspond to the *E* and *H* modes, respectively.

rial should cease to exhibit the corresponding properties.

The process of forming ferroelectric spherical particles involves a certain error. This error arises primarily in the permittivity of the material and also in the process of treating and forming resonators, i.e., in their geometric sizes. We can assess the admissible error from the formula for the electrical size of spherical particles:

$$\rho = \frac{k_1 a}{N}, \tag{3}$$

where a is the radius of the particle, $k = \omega\sqrt{\varepsilon_0\varepsilon_1\mu_1}$ is the wave vector, and $N = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$. Expression (3) can be rewritten as follows:

$$f = \frac{N\rho}{2\pi a\sqrt{\varepsilon_0\varepsilon_1\mu_1}}, \tag{4}$$

where f is the frequency of electromagnetic oscillations. In this expression, the variables are the radius of the particle a and its relative permittivity ε_1. The electrical size ρ is determined by the resonance frequency for electric or magnetic modes.

The dependences of the resonance frequency on the radius of particles and on their permittivity are presented in Figs. 5 and 6, respectively. If we suggest that random deviations from the required value are possible toward both larger and smaller values, we can assume that these deviations should cause a frequency drift of no larger than the half-width of the range determined by the curve in Fig. 4. Thus, for spherical particles of radii 0.5 and 1.0 mm at an operating frequency of 10 GHz, the half-width of the operating frequency range with a negative dispersion will be equal to 16 MHz. This means that, for example, for spherical particles with $\varepsilon_1 = 400$ and $\tan\delta = 0.003$ with the resonance frequency $f = 10$ GHz, the random deviation from the required radius $r = 1$ mm should not exceed 1.5 μm. Similar high-accuracy requirements are imposed on the permittivity itself: for the given example, the permittivity should be accurate to within 400 ± 3.

4. EXPERIMENTAL INVESTIGATION

In order to verify the validity of the above-described theory, we prepared a series of samples from a ferroelectric ceramic material based on $Ba_xSr_{1-x}TiO_3$ [18] with the parameters $\varepsilon = 378$ and $\tan\delta \approx 0.004$. The samples were prepared from the material in the form of pellets that were cut into small cubes and, then, further treated with abrasives to attain a spherical form and required sizes. The permittivity is characterized by an inhomogeneous distribution over the volume of the

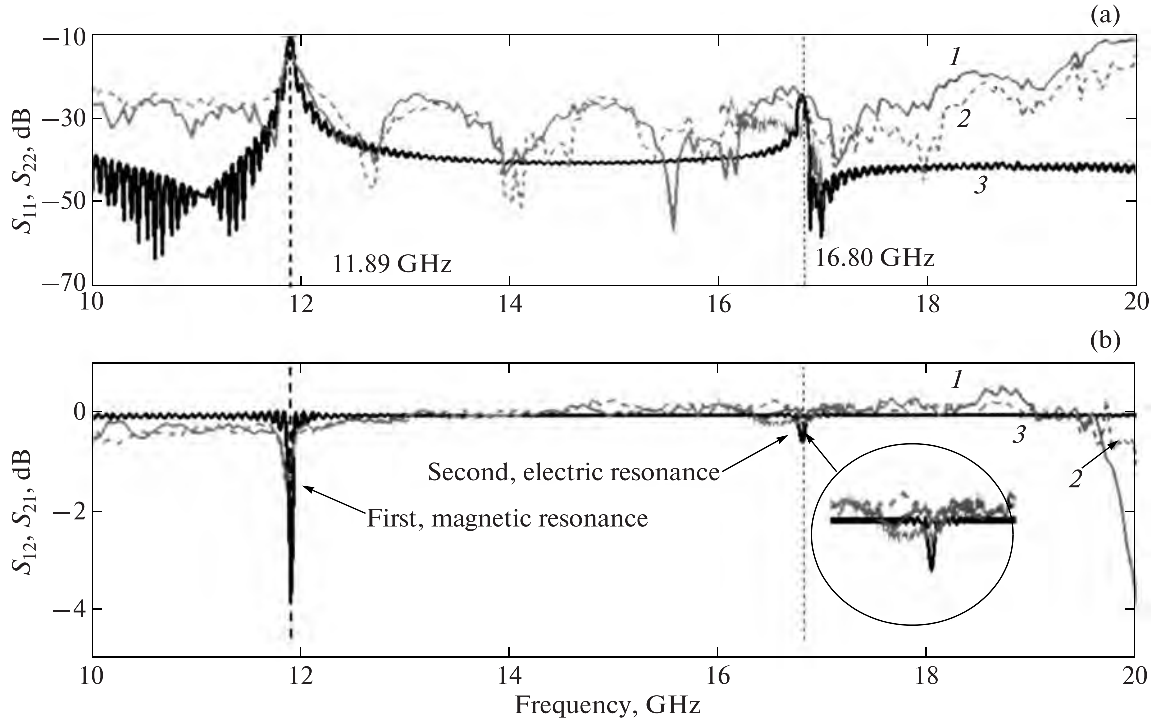

Fig. 7. Results of the experimental investigation of the ferroelectric ceramic spherical resonator. (a) Refractive indices for the spherical resonator in the waveguide: the measurement data for (*1*) S_{11} and (*2*) S_{22} and computer simulation data for (*3*) $S_{11} = S_{22}$. (b) Transmission coefficients for the spherical resonator in the waveguide: measurement data for (*1*) S_{12} and (*2*) S_{21} and computer simulation data for (*3*) $S_{12} = S_{21}$.

ceramic material. So, the optimum method to prepare a few samples with close resonance frequencies is to individually select appropriate sizes specifically for each sample.

The experimental investigation of the resonant properties of a spherical dielectric resonator was carried out in a rectangular waveguide [19]. The results of measurements for one of the samples are presented in Fig. 7 in comparison with the results of the calculation. The magnetic resonance corresponding to the fundamental mode in a dielectric sphere (f_1 = 11.89 GHz) is clearly pronounced as a resonance peak, whereas the electric resonance (f_2 = 16.8 GHz) is considerably less pronounced. The resonant dielectric sphere at the frequency f_1 forms a magnetic dipole and determines the negative effective magnetic permeability of the medium, and at the frequency f_2, there arises an electric dipole that is responsible for the negative permittivity of the medium. A weakly pronounced resonance response of the electric dipole should be expected to lead to a smaller contribution from these resonators to the negative permittivity of the metamaterial on the lattice of dielectric resonators as compared to the effective permeability.

The numerical electrodynamic simulation has demonstrated that the array of magnetic dipoles on the spheres with magnetic resonance behaves like a medium with a negative effective permeability and prevents wave propagation in the resonance region, whereas the array of electric dipoles on the spheres with electric resonance exhibits weak resonant properties and the wave propagates through this medium with only a slight attenuation. In this regard, a model has been advanced for a metamaterial composed of identical dielectric spherical resonators with magnetic resonance, where excitation of an electric dipole should be achieved through the use of coupling between the resonators.

5. CONCLUSIONS

The investigation of metamaterials for the purpose of their application in superhigh-frequency (SHF) devices has shown much promise and has been an interesting direction of scientific research. A bulk isotropic metamaterial can be prepared using spherical dielectric resonators arranged into a cubic structure. A medium with a negative dispersion can be prepared with the use of two types of resonators that are located in the immediate vicinity and interact with each other. A ferroelectric ceramic material, which exhibits a high permittivity and a low level of SHF loss, is a reasonable choice of the material for manufacturing dielectric particles. To attain the required properties of the designed metamaterial, high requirements should be imposed on the accuracy in fabricating ceramic resonator constituents of the metamaterial. An optimum method is to select required samples with the desired resonance frequency from the total number of produced resonators.

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